Theoretical Analyses of Anneal Offsets and Reverse Annealing

Hidetoshi Nishimori

with

Yuki Susa, Yu Yamashiro, Masaki Ohkuwa, Masayuki Yamamoto

Tokyo Institute of Technology

Itay Hen and Daniel Lidar

University of Southern California

Outline

1. Quantum annealing: Recap

2. Quantum annealing with non-stoquastic Hamiltonians

3. Analytical theory of quantum annealing with inhomogeneous field-driving
   ← Anneal offsets

4. Analytical theory of quantum annealing with reverse annealing

Recapitulation of quantum annealing

Goal : Combinatorial optimization problems

→ Ground-state search of the Ising model

Given $\{J_{ij}\}$ and $\{h_i\}$, find the values of variables $\{\sigma_i^z\}$ to minimize H_0

$$H_0 = -\sum_{i,j} J_{ij}\sigma_i^z\sigma_j^z - \sum_{i=1}^{N} h_i\sigma_i^z, \quad (\sigma_i^z = \pm 1)$$

Use quantum fluctuations to search for the solution.

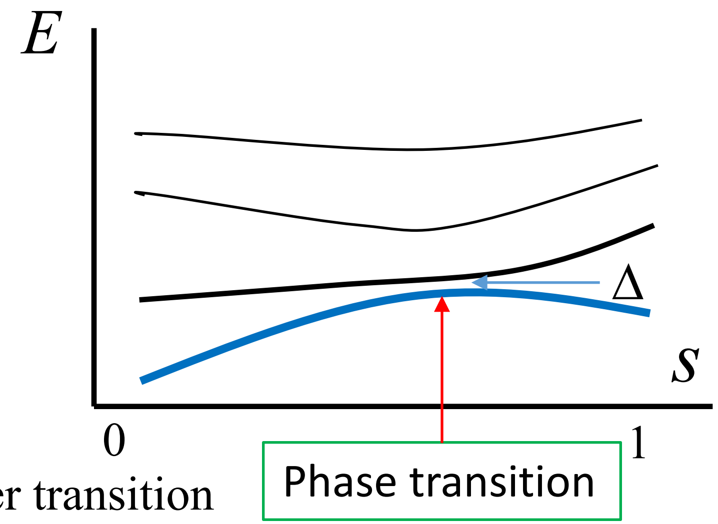
$$H = sH_0 - (1-s)\sum_{i=1}^{N}\sigma_i^x \quad (s : 0 \rightarrow 1)$$

Computational complexity

Pure quantum dynamics of closed system

Adiabatic theorem

$\frac{1}{\Delta^2}$



Gap scaling

e^{aN} 1st order transition

N^{b} 2nd order transition

Complexity

e^{2aN} (hard)

N^{2b} (easy)

Very important to avoid 1st order transition

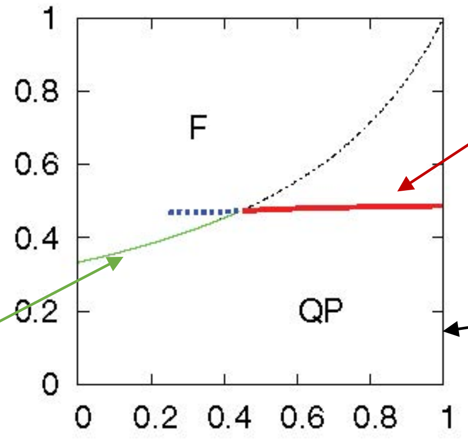
Non-stoquastic Hamiltonian

$$H = s\lambda H_0 - (1-s)\sum_{i=1}^{N}\sigma_i^x + s(1-\lambda)N\left(\frac{1}{N}\sum_{i=1}^{N}\sigma_i^x\right)^p$$

$$\exp(-\beta H)$$

Impossible to simulate classically (sign problem). “Strong” quantum effects.

$$H_0 = -N\left(\frac{1}{N}\sum_i \sigma_i^z\right)^p$$



1st order transition
(Exponential time) $\tau \propto e^{aN}$

2nd order transition
Polynomial time $\tau \propto N^b$

Conventional method

Non-stoquasticity leads to an exponential speedup, not just impossibility of simulation, as compared to conventional quantum annealing

*Frontiers in ICT, **4**, 2 (2017), PRE **85**, 051112 (2012), J. Phys. A **48**, 335301 (2015)*

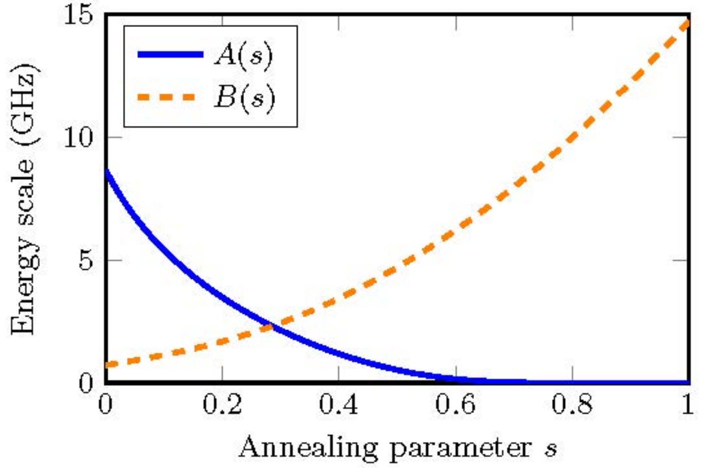
Inhomogeneous driving of the transverse field

Anneal offsets

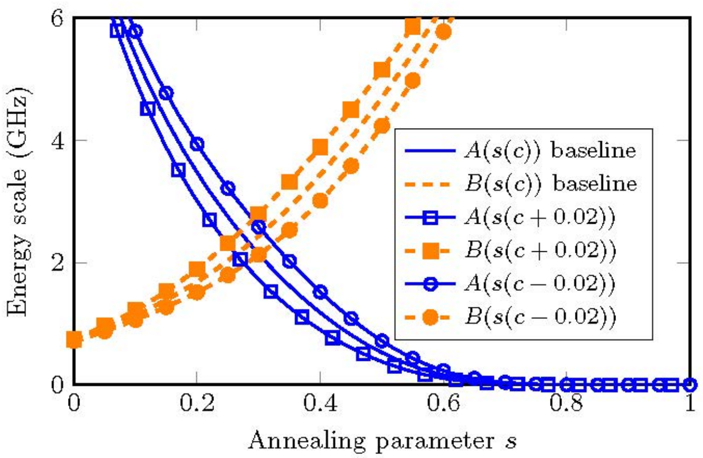
Anneal offsets: Individual (qubit-dependent) control of the transverse field and the cost function.

$$H(s) = -\sum_i A_i(s)\sigma_i^x + \sum_i B_i(s)h_i\sigma_i^z + \sum_{i<j}\sqrt{B_i(s)B_j(s)}J_{ij}\sigma_i^z\sigma_j^z$$

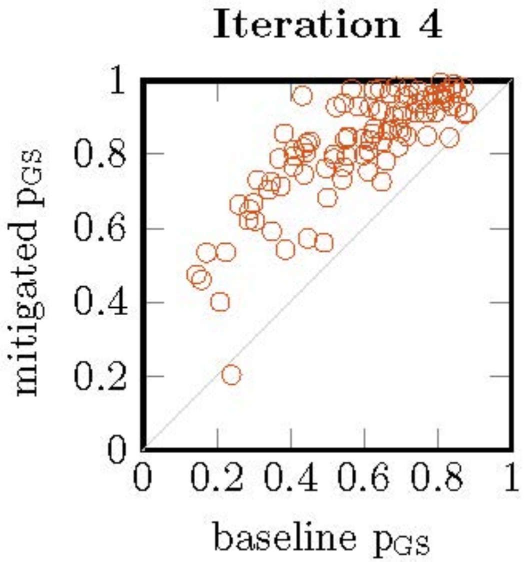
Lanting et al, Phys. Rev. A (2017)



Without offset



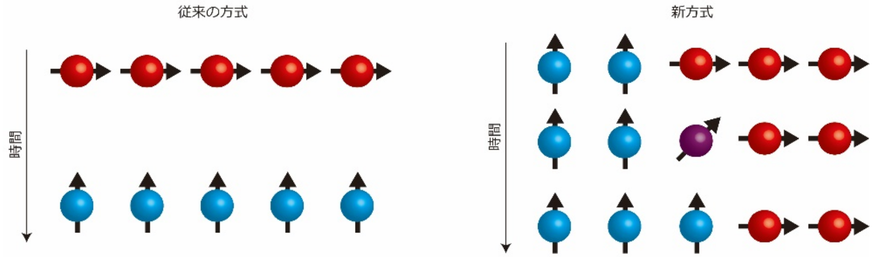
With offset



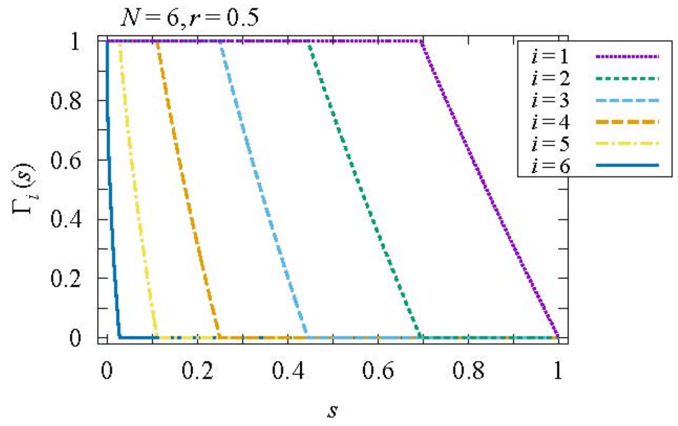
Increased success probability

Theory for inhomogeneous transverse field

$$H = sH_0 - (1-s)\sum_{i=1}^{N}\sigma_i^x$$



$$H = sH_0 - (1-s)\sum_{i=1}^{N(1-\tau)}\sigma_i^x - 0\sum_{i=N(1-\tau)+1}^{N}\sigma_i^x$$



$$H_0 = -N\left(\frac{1}{N}\sum_{i=1}^{N}\sigma_i^z\right)^p - \sum_{i=1}^{N}h_i\sigma_i^z$$

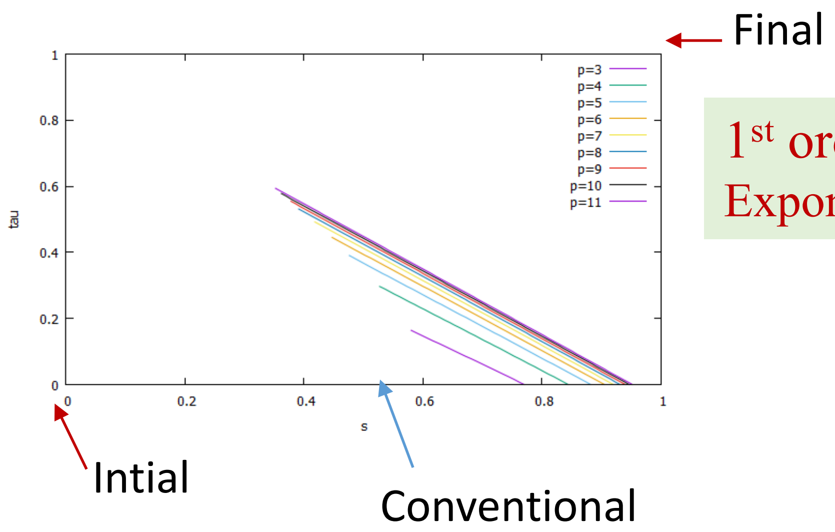
Random-longitudinal-field Ising model, for which non-stoquastic method doesn't work

Dickson and Amin, Phys. Rev. A (2012), Rams et al, New J. Phys. (2016)

Result

Ideal case of isolated quantum dynamics

$$H = sH_0 - (1-s)\sum_{i=1}^{N(1-\tau)} \sigma_i^x - 0 \sum_{i=N(1-\tau)+1}^{N} \sigma_i^x \qquad H_0 = -N\left(\frac{1}{N}\sum_{i=1}^{N}\sigma_i^z\right)^p - \sum_{i=1}^{N} h_i\sigma_i^z$$



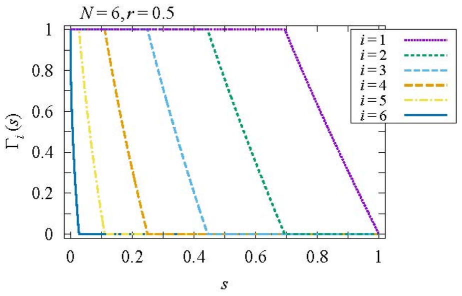
1st order phase transition disappears!
Exponential speedup by a simple inhomogeneous control of the transverse field.

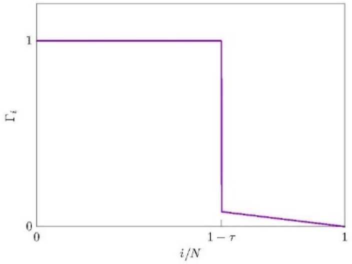
Susa, Yamashiro, Yamamoto and Nishimori, J. Phys. Soc. Jpn. ***87****, 023002 (2018)*

Non-ideal case

Field strength as a function of time

Original





Zero at the end

Result: Another first-order transition line. But reduced width of barrier.

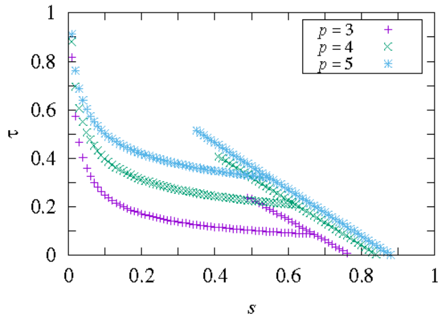


Figure 4: Phase diagram of QA at $T = 0.01$. Lines of first-order transitions appear at the lower left of the phase diagram in addition to the straight lines that exist at $T = 0$.

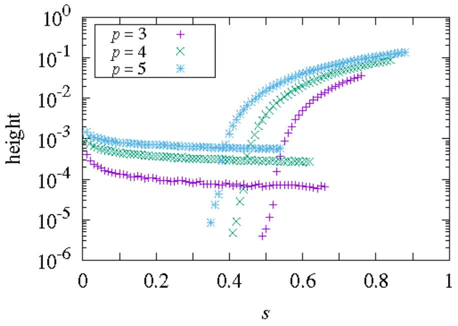


Figure 5: Height of free energy barrier separating two minima at first-order phase transitions at $T = 0.01$ for QA.

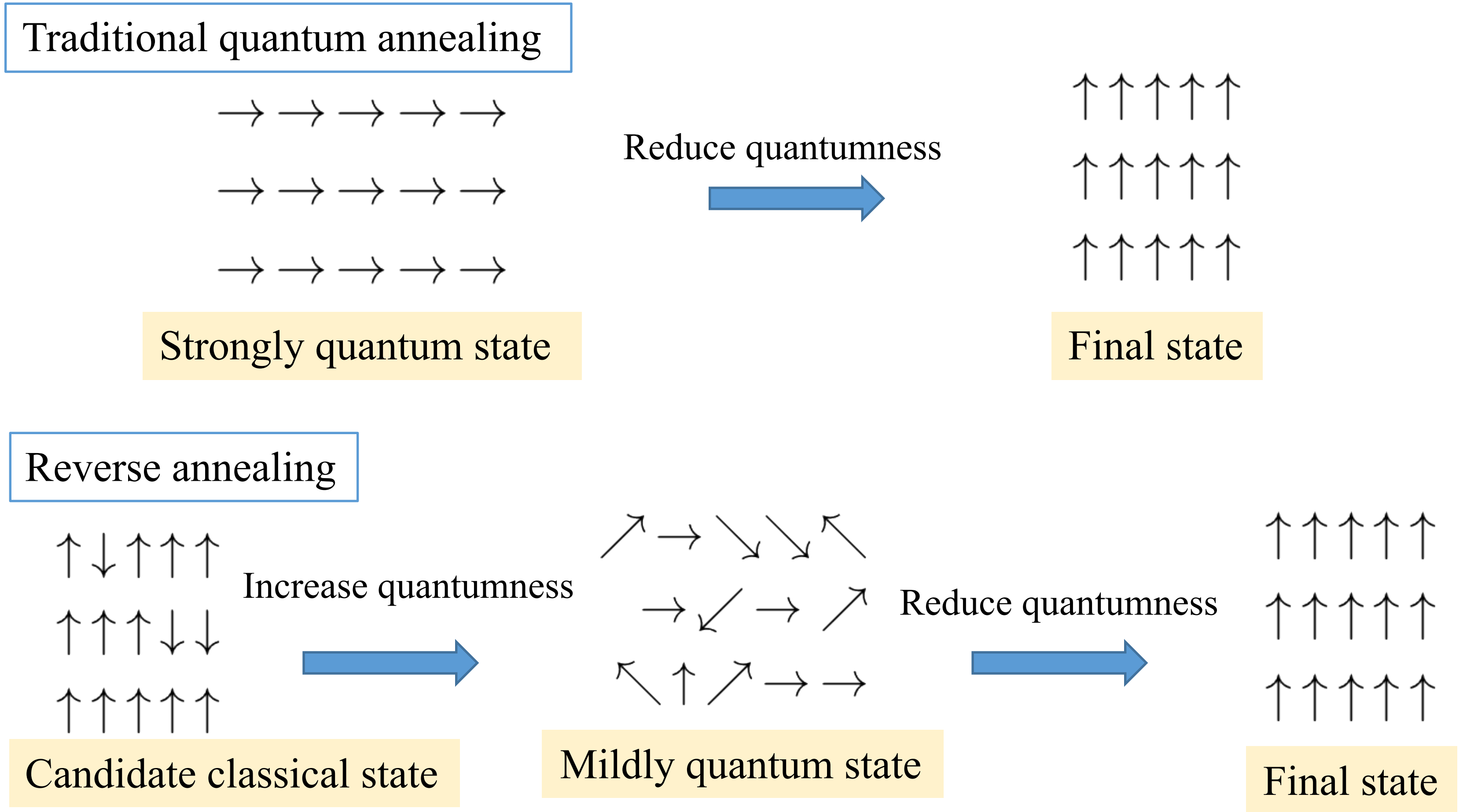
Classical simulated annealing with inhomogeneous temperature drive

Assign local (inverse) temperature to each site and increase each of them one by one.

$$H = -N\left(\frac{1}{N}\sum_{i=1}^{N}\beta_i\sigma_i\right)^p - \sum_{i=1}^{N}h_i\sigma_i \qquad \beta = 1/T$$

- First-order transition persists.
- To be contrasted with the quantum case: inhomogeneous transverse field erased the first order transition.
- One of rare examples where quantum approach is explicitly shown to be better than the corresponding classical approach. "Limited quantum speedup"

Reverse annealing



Perdomo-Ortiz et al (2010)

Mean-field theory of reverse annealing

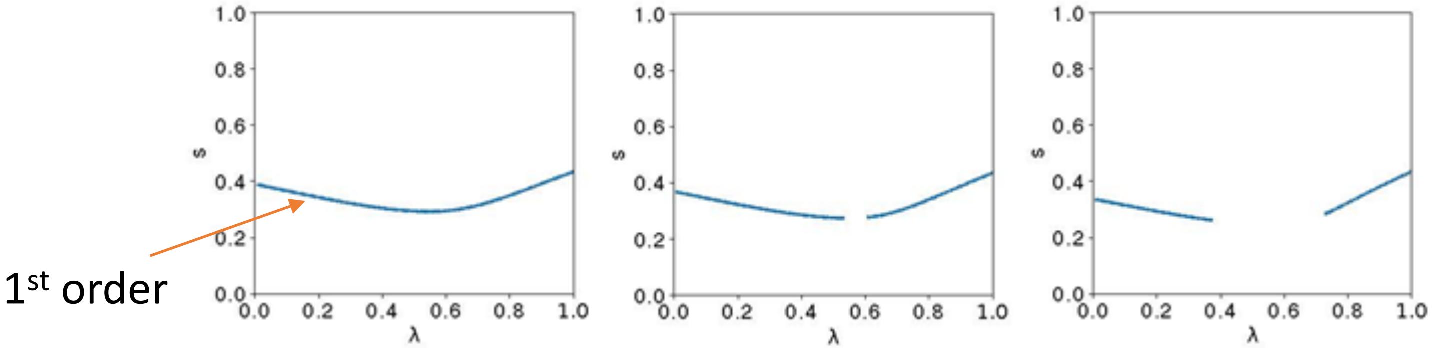
$$H = sH_0 + (1-\lambda)(1-s)H_{\mathrm{init}} + (1-s)\lambda H_{\mathrm{TF}}$$

$$H_{\mathrm{init}} = -\sum_{i=1}^{N} \epsilon_i \sigma_i^z \quad (\epsilon_i = 1 \text{ or } -1)$$

Start from the classical state ε_i and then increase quantum fluctuations by H_{TF}

$$s = \lambda = 0 \longrightarrow s = \lambda = 1$$

$$H_0 = -N\left(\frac{1}{N}\sum_{i=1}^{N}\sigma_i^z\right)^p - \sum_{i=1}^{N} h_i \sigma_i^z$$



Initial state closer to the correct state

This method exponentially accelerates QA.

Conclusion

1. Quantum annealing with non-stoquastic Hamiltonian

2. Quantum annealing with inhomogeneous field driving

3. Quantum annealing with reverse annealing

- Exponential speedup *in comparison with the conventional quantum annealing.* 1st order → 2nd order / no transition.
- Also *in comparison with the corresponding classical simulated annealing* for 2.