Optimizing Quantum Annealing Performance via Quantum Control

Qubits 2018: D-Wave Users Conference

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Overview

Problem
Develop a method to optimize control schedules for general adiabatic quantum computation (AQC) algorithms that is

• Scalable/Efficient: Convergence rates that do not depend on system size
• Practical: does not require knowledge of energy spectrum or computational solution
• Robust: robust to system uncertainty, e.g., noise
Overview

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Develop a method to optimize control schedules for general adiabatic quantum computation (AQC) algorithms that is
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Motivation
• Optimized control can facilitate computational speedup
  - Boundary cancellation methods (Rezakhani et. al. PRA 2011)
• Many techniques are not practical
  - Require knowledge of the instantaneous energy spectrum (Zeng et al JPA 2016)
  - Require knowledge of the computational solution (Brif at. AI NJP 2014)
  - Not robust to system uncertainty (Roland, Cerf PRA 2002, Rezakhani et al. PRL 2009)
Quantum Control

Objective
Perform particular quantum operation with high fidelity, potentially while simultaneously mitigating the effects of unwanted environment interactions

Open Loop Control
- Offline Optimized control path
- Optimized state w.r.t. some observable
- Relies on system model
- May/may not be robust to uncertainty
- Example: optimal control, robust control

Closed Loop Control
- Active or Iteratively Optimized control path
- Optimized state w.r.t. some observable
- Inherently robust to system uncertainty
- Requires intermediate or terminal measurement of an observable

Feedback (intermediate or terminal)
Quantum Control for Adiabatic Quantum Computation

Local adiabatic control (LAC)
• Relies on “instantaneous adiabatic theorem”
  - satisfy the adiabatic condition at each instance in time
• Minimizes the time needed to reach the adiabatic regime based on the rate of change of the evolution

Boundary cancellation control (BCC)
• Relies on “final time adiabatic theorem”
  - Minimizes error in the adiabatic approximation
  - Polynomial error improvement of LAC by setting the first $n - 1$ derivatives of the Hamiltonian to zero at the boundaries

$LAC$ path

$BCC$ path
Closed Loop Control Protocol

Closed Loop Optimized Adiabatic Quantum Control (CLOAQC)

In situ control protocol designed to minimize system energy via Simultaneous Perturbation Stochastic Approximation (SPSA) optimization (Spall 1992)

Build control schedules for SPSA gradient

Initialize control schedule

$\Lambda^+_k$  $\Lambda^-_k$  QPU
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Build control schedules for SPSA gradient

\[ \Lambda^+_k \]
\[ \Lambda^-_k \]

QPU

\[ |\psi_1(T)\rangle \]
\[ |\psi_2(T)\rangle \]
\[ \vdots \]
\[ |\psi_M(T)\rangle \]

\[ E_1(\Lambda_k) \]
\[ E_2(\Lambda_k) \]
\[ \vdots \]
\[ E_M(\Lambda_k) \]

M samples of system energy
Closed Loop Control Protocol

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Build control schedules for SPSA gradient

Initialize control schedule

$\Lambda_k^+$

$\Lambda_k^-$

$\Lambda_{k+1}^+$

$\Lambda_{k+1}^-$

QPU

$|\psi_1(T)\rangle$

$|\psi_2(T)\rangle$

$|\psi_M(T)\rangle$

$E_1(\Lambda_k)$

$E_2(\Lambda_k)$

$\vdots$

$E_M(\Lambda_k)$

$g_k = \frac{\hat{E}(\Lambda_k^+) - \hat{E}(\Lambda_k^-)}{2\beta_k}$

$\Lambda_{k+1} = \Lambda_k + \alpha_k (g_k + \nabla J_{ad})$

Adiabaticity Constraint

$J_{ad}(\Lambda) = \lambda \sum_{\mu=A,B} \int_0^T \|\dot{\mu}(t, \Lambda)\|^2 dt$

$21$ September $2018$
Closed Loop Control Protocol

Closed Loop Optimized Adiabatic Quantum Control (CLOAQC)

*In situ* control protocol designed to minimize system energy via Simultaneous Perturbation Stochastic Approximation (SPSA) optimization (Spall 1992)

**Initial control schedule**

- \( \Lambda^+_k \)
- \( \Lambda^-_k \)

**Build control schedules for SPSA gradient**

- \( \Lambda^+_k \)
- \( \Lambda^-_k \)
- \( \Lambda^+_k+1 \)
- \( \Lambda^-_k+1 \)

**QPU**

**M samples of system energy**

- \( |\psi_1(T)\rangle \)
- \( |\psi_2(T)\rangle \)
- \( |\psi_M(T)\rangle \)

**Update gradient and control schedules**

\[
g_k = \frac{\hat{E}(\Lambda^+_k) - \hat{E}(\Lambda^-_k)}{2\beta_k}
\]

**Control Update**

\[
\Lambda_{k+1} = \Lambda_k + \alpha_k (g_k + \nabla J_{ad})
\]

**Adiabaticity Constraint**

\[
J_{ad}(\Lambda) = \lambda \sum_{\mu=A,B} \int_0^T \|\dot{\mu}(t, \Lambda)\|^2 dt
\]

**Req. Experiments:** \(2KM\)
CLOAQC: Numerical Study
Grover’s Search Problem

Hamiltonian

$$H_{ad}(s) = A(s)[I - |+\rangle\langle+|] + B(s)[I - |m\rangle\langle m|]$$

Controls

$$A(s) = \sum_{i=0}^{d-1} a_i s^i, B(t) = \sum_{i=0}^{d-1} b_i s^i$$

Trace Distance

$$D = \sqrt{1 - |\langle \Phi_0(1) | \psi(1) \rangle|^2}$$

CLOAQC converges to known LAC solutions
Hamiltonian
\[ H_{ad}(s) = A(s)[|I\rangle\langle I|] + B(s)[|I\rangle\langle m| + |m\rangle\langle I|] \]

Controls
\[ A(s) = \sum_{i=0}^{d-1} a_i s^i, \quad B(s) = \sum_{i=0}^{d-1} b_i s^i \]

Trace Distance
\[ D = \sqrt{1 - |\langle \Phi_0(1) | \psi(1) \rangle|^2} \]

CLOAQC converges to known LAC solutions
CLOAQC: Numerical Study
MAX 2-SAT

**Problem:** Determine maximum number of satisfying assignments for a Boolean formula

\[ F[\{x_i\}_{i=1}^N] = (x_3 \lor x_1) \land (\neg x_5 \lor x_2) \land \cdots \land (x_4 \lor \neg x_3) \]

\[ H_P = \sum_{k=1}^{M} H_{C_k}, \quad H_{C_k} = \left( \frac{1 - v_{x_i}^k \sigma_i^Z}{2} \right) \left( \frac{1 - v_{x_j}^k \sigma_j^Z}{2} \right) \]

**AQC Hamiltonian**

\[ H(t) = A_1(t) \sum_i \sigma_i^X + A_2(t) \sum_{i \neq j} \sigma_i^X \sigma_j^X + B_1(t) \sum_i h_i \sigma_i^Z + B_2(t) \sum_{i \neq j} J_{ij} \sigma_i^Z \sigma_j^Z \]

Increasing control DOF leads to improvements in computational accuracy and enhancements in minimum gap.
CLOAQC: Numerical Study
Robustness to Noise

Grover with unitary control errors

\[ H_{ad}'(s) = H_{ad}(s) + H_E(s) \]
\[ H_{ad}(s) = A(s)[I - |+\rangle\langle+|] + B(s)[I - |m\rangle\langle m|] \]
\[ H_E(s) = \Gamma(s) \sum_i \hat{m}_i \cdot \hat{\sigma}_i \]

Error Scenarios

a) \( \Gamma(s) = Cs \)

b) \( \Gamma(s) = C \sin(\pi s) \)

c) \( \Gamma(s) = \frac{1}{2} \sin(C\pi s) \)

CLOAQC exhibits robustness to sufficiently small and slow-oscillating unitary control errors
Control Capabilities on the D-Wave QPU

**2000Q System**
- Allows for *some* control over annealing path
- Path must be monotonic
- New features
  - Pausing
  - Quenching
- Permits experimental testing of CLOAQC!
Content Addressable Memory Problem

Traditional Memory

• Input is address location of the desired content
• Output is the content of the address

Content Addressable Memory (CAM)

• Input is content of the stored memory
• Output is the location of the desired content
Quantum CAM

Problem Design
Cast CAM problem as an adiabatic quantum optimization problem

Keys: $K = [k^{(1)}, k^{(2)}, ..., k^{(m)}]^T$
Values: $V = [v^{(1)}, v^{(2)}, ..., v^{(m)}]^T$

Hamiltonian Description

$$H(t, \theta) = A(t)H_X + B(t)H_\theta$$

$$H_X = -\sum_i^n \sigma_i^X$$

$$H_\theta = -\sum_{i,j} w_{ij} \sigma_i^Z \sigma_j^Z - \sum_i^n \theta_i v_i^{(0)} \sigma_i^Z$$

Hebbs Learning Rule

$$W = \begin{pmatrix} 0 & W_B \\ W_B^T & 0 \end{pmatrix}$$

$$W_B = \frac{1}{n} K^T V$$

Maximum Classical Learning Capacity: $C(n) = \frac{n}{2} \log(n)$

Santa et al. PRA 20017
Schrock et al, Entropy 2017
QCAM Preliminary Experimental Results
CLOAQC Convergence Scaling

Problem Description
• $n = 16$ logical qubits
• # encoded memories: $m = 0.2n$
• $1 \mu s$ annealing time
• $N = 1000$ annealing runs
• 20 realizations of CLOAQC
• 500 iterations of CLOAQC

Convergence Scaling
Convergence Rate: $O(k^\beta)$

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<th>$\theta$</th>
<th>$\beta$</th>
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<tr>
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</table>

Fidelity
$$ F = \frac{1}{N} \sum_{i}^{N} \delta_{c_i,n} $$

Single Instance Convergence Scaling
QCAM Preliminary Experimental Results
Fidelity vs. Bias

Problem Description
• \( n = 16 \) logical qubits
• \( m = 0.2n \) encoded memories
• 1 \( \mu s \) annealing time
• \( N = 1000 \) annealing runs
• 20 realizations of CLOAQC
• 500 iterations of CLOAQC

Fidelity
\[
F = \frac{1}{N} \sum_{i}^{N} \delta_{c_i,n}
\]
**QCAM Preliminary Experimental Results**

**Fidelity vs. Problem Size**

### Problem Description
- Number of logical qubits $n = 8, 16, 32, 64$
- Bias $\theta = 0.1$
- # stored memories: $m = 0.2n$
- 1 $\mu$s annealing time
- $N = 1000$ annealing runs
- 20 realizations of CLOAQQC
- 500 iterations of CLOAQQC

### Fidelity

$$F = \frac{1}{N} \sum_{i}^{N} \delta_{c_i, n}$$

![Median Infidelity vs. System Size](chart)
Summary

Conclusions

• CLOAQC can be used to improve computational accuracy of the D-Wave QPU
• Encouraging preliminary results suggest QCAM recall accuracy can be improved by CLOAQC

Future Work

• Explore benefits of control for QCAM capacity
• Optimizing control with respect to capacity
• Methods for accelerating CLOAQC convergence