Simulations of the Ising model on a Shastry-Sutherland lattice by quantum annealing

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This work was supported by the Department of Energy, Office of Science Early Career Research Program and Basic Energy Sciences program office.
We demonstrate how quantum annealing enables accurate simulations of many-particle Hamiltonian systems.
Rare-earth tetraboride materials

Can we simulate these behaviors using quantum annealing?
The Shastry-Sutherland lattice

Ising magnet

\[ H = J_1 \sum_{\langle ij \rangle} \sigma^z_i \sigma^z_j + J_2 \sum_{\langle ij \rangle} \sigma^z_i \sigma^z_j \]

\[ J_2 = \text{Dimer} \]

\[ J_1 = \text{Square} \]

\[ \leftrightarrow \quad \text{RB}_4, \quad R = \text{La-Lu} \]
Phase diagram

Dublenych 2012
D-Wave 2000Q Processor

Coupled RF-SQUID Qubits

\[ H = A(s) \sum_i \sigma_i^x + B(s) \left[ \sum_i h_i \sigma_i^z + \sum_{\langle ij \rangle} J_{ij} \sigma_i^z \sigma_j^z \right] \]

Independently tunable biases and couplers

Chimera Topology
Embedding the Shastry-Sutherland Lattice

- Half-cell embedding with 500 logical qubits, 2000 physical
Self-consistent mean-field boundary conditions

\[ H = \sum_i h_i^z \sigma_i^z + h^z \sum_i \sigma_i^z + \sum_{\langle ij \rangle} J_{ij} \sigma_i^z \sigma_j^z \]

minimize \( \langle m \rangle - \langle m \rangle \) 

subject to \( \text{sgn}(h_i^z) = \text{sgn}(h^z) \) \( \forall i \)

Independent tunability of biases allows us to explore novel boundary conditions
Forward Annealing

\[ H = A(s) \sum_i \sigma_i^x + B(s) \left[ \sum_i h_i \sigma_i^z + \sum_{\langle ij \rangle} J_{ij} \sigma_i^z \sigma_j^z \right] \]

Adiabatic evolution and readout
Forward Anneal with the half-cell embedding
Reverse Annealing

\[ H = A(s) \sum_i \sigma_i^x + B(s) \left[ \sum_i h_i \sigma_i^z + \sum_{\langle ij \rangle} J_{ij} \sigma_i^z \sigma_j^z \right] \]

Diagram showing the process of reverse annealing with reverse evolution, pause, and forward evolution. The equation describes the Hamiltonian used in the process.

- Reverse evolution
- Pause
- Forward evolution

The graph illustrates the relationship between frequency and temperature, with labels for Begin + End and A(s) << B(s) indicating the initial and final conditions.
Quantum Evolution Markov Chain

\[ H = A(s) \sum_i \sigma^x_i + B(s) \left[ \sum_i h_i \sigma^z_i + \sum_{\langle ij \rangle} J_{ij} \sigma^z_i \sigma^z_j \right] \]

- Iterative reverse annealing schedule

\[ \text{Begin + End} \quad A(s) \ll B(s) \]
Quantum evolution Markov chain

\[ H = A(s) \sum_i \sigma_i^x + B(s) \left[ \sum_i h_i \sigma_i^z + \sum_{\langle ij \rangle} J_{ij} \sigma_i^z \sigma_j^z \right] \]

- Iterative reverse annealing schedule

- \( A(s) \ll B(s) \) Begin + End

![Graph showing frequency vs. parameter s with labels and trends]

![Graph showing magnetization vs. iterations with various s values]
QEMC Motif Convergence

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QEMC Structure Factor convergence
Quantum evolution Markov chain: half-cell embedding
Quantum evolution Markov chain: half-cell embedding
Quantum evolution Markov chain: half-cell embedding
Quantum evolution Markov chain: half-cell embedding
Simulations of the Ising model on a Shastry-Sutherland lattice by quantum annealing

- We demonstrate how quantum annealing enables accurate simulations of many-particle Hamiltonian systems.
- We sample the ground state energy configurations of the the SS Ising model to calculate the structure factor
- We develop a novel method for mitigating finite size and defects.
- We observe good agreement between the observed and expected material behaviors.
- We can now explore defect physics and temperature effects in this model.
References


Quantum evolution Markov chain without boundary conditions
Forward Anneal - Tilt Embedding
Boundary Refinement

Boundary Field Refinement

Magnetic bias, $h$

Iterations
Quantum evolution Markov chain: half-cell embedding